Definition of a Function:

Function: each input has exactly ________ output.

Vertical Line Test:

Important Characteristics of a Function:

Increasing: x-interval (from left to right) where a function is going uphill

Decreasing: x-interval (from left to right) where a function is going downhill

Constant function: x-interval (from left to right) where a function is horizontal

Relative max: a point where a function changes from increasing to decreasing; a "mountain-top"

Relative min: a point where a function changes from decreasing to increasing; a "mountain-top"

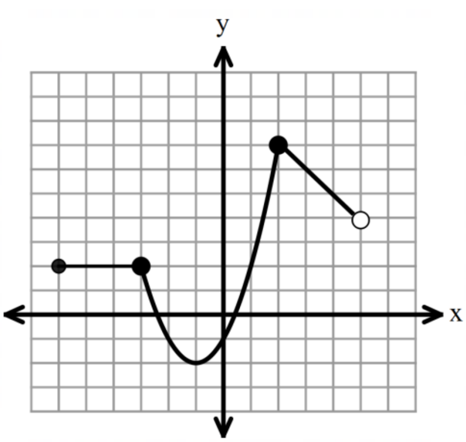
Domain: set of all inputs (x-values) for which a function is defined

Range: set of all outputs (y-vaues) for which a function is defined

Reminder: interval notation

- Use [brackets] for closed circles and (parenthesis) for open circles
  - Exception: we will use (parenthesis) for *all* increasing/decreasing intervals
  - Some textbooks use brackets instead of parenthesis.
- For x-values, use ***left, right***
- For y-values, use ***bottom, top***
- For undefined points, such as open circles, use a parenthesis instead of a bracket.

Example 1: Identify the important characteristics of the graph shown.



Is the graph a function? How do you know?

Interval(s) increasing

Interval(s) decreasing

Relative max, if any

Relative min, if any

Domain

Range

Odd and Even Functions:

A function is **even** if $f(-x) = f(x)$ for all values of x.

A function is **odd** if $f(-x) = -f(x)$ for all values of x.

Examples 2 – 4: Determine if the function is even, odd, or neither.

2) $f(x) = 3x^3 + x^2 - 4$

3) $g(x) = 4x^2 + x^4$

4) $h(x) = -2x^5 + x^3$

For #5 – 7, evaluate each function at the given value of the independent variable.

Given that $f(x) = \begin{cases} 5x - 3 \text{ if } x < -4 \\ 3x - 5 \text{ if } x \geq -4 \end{cases}$ and $g(x) = 2x^2 - 6$

5) $f(-3)$

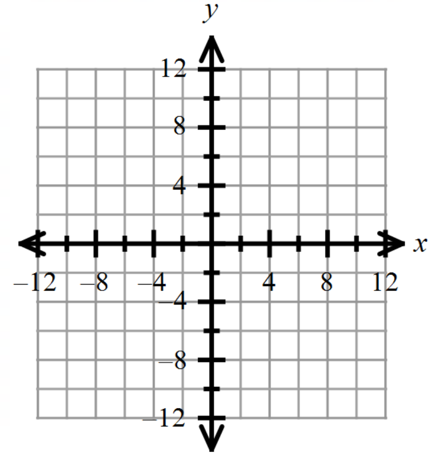
6) $f(-8)$

7) $g(x - 1)$

8) Given that $f(x) = x^2 + 9x - 7$, find $\frac{f(x+h)-f(x)}{h}$ if $x \neq 0$. Simplify your answer.

9) Graph $f(x)$ on the provided coordinate system.

$$f(x) = \begin{cases} x + 5 \text{ if } -8 \le x < 2 \\ -4 \text{ if } x = 2 \\ -x + 5 \text{ if } x > 2 \end{cases}$$



10) Write the equation of the line in point-slope form that passes through $(3, -6)$ has an x-intercept of -2. Also write your answer in (h, k) form and slope-intercept form.

For #11 – 13: Perform the indicated operations for the given functions for f and g.

11) Find $\frac{f}{g}$ if $f(x) = 4x^2 - 7x$ and $g(x) = x^2 - 5x - 14$. Factor your answer completely.

12) Find $f - g$ if $f(x) = 7x - 9$ and $g(x) = 2x - 4$.

13) Find $g(f(x))$… also written as $(g \circ f)(x)$… if $f(x) = -4x + 8$ and $g(x) = 3x + 5$.

Inverses of Functions:

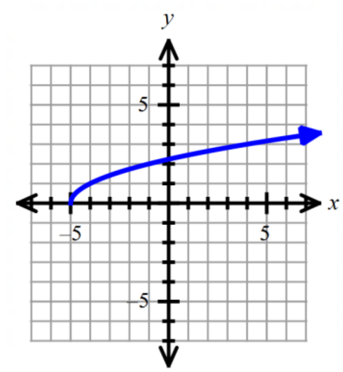
- To find the inverse of a function: exchange the x and y-variables, and solve for y.
- Given that f and g are inverse functions, and if $f(a) = b$, then $g(b) = a$.
  - In other words, each ordered pair has the x- and y-coordinates exchanged.
- Given that f and g are inverse functions, then $f(x)$ and $g(x)$ are reflections in the line $y = x$.
- Given that f and g are inverse functions, then $f(g(x)) = x$ and $g(f(x)) = x$.

If a function is one-to-one, then it passes both the Vertical Line Test and the Horizontal Line Test.

- This means that both the function and its inverses are functions.
- For each input, there is exactly one output. And for each output, there is exactly one input.

14) Find the equation of the inverse of the given one-to-one function: $g(x) = \frac{5}{7x-8}$

15) Use the graph of $f(x)$, as shown to the right, to draw the graph of its inverse function.



16) Determine which two functions below are inverses of each other.

$f(x) = 3x$ $\qquad$ $g(x) = \frac{x}{3}$ $\qquad$ $h(x) = \frac{3}{x}$

Transformations of a Function

Given $f(x)$, then $g(x) = a \cdot f(x - h) + k$ has the following transformations on f.

- If $|a| > 1$, then there is a vertical stretch by a factor of a.
- If $|a| < 1$, then there is a vertical compression by a factor of a.
- If $a < 0$, then there is a vertical reflection.
- Horizontal shift: h units
- Vertical shift: k units

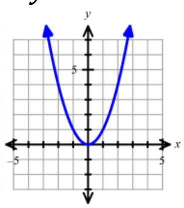
You can always use a table of values to help with a transformation!

Parent Functions

Students need to know the basic form of each parent function below.

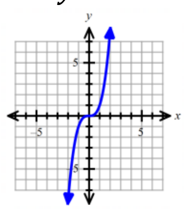
Quadratic

$y = x^2$



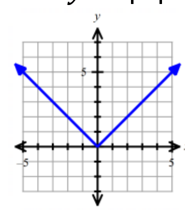
Cubic

$y = x^3$



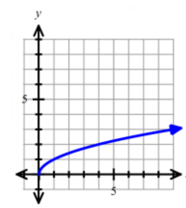
Absolute Value

$y = |x|$



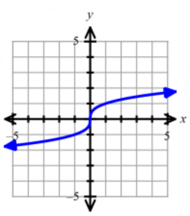
Square Root

$y = \sqrt{x}$



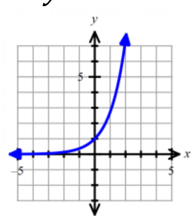
Cube Root

$y = \sqrt[3]{x}$



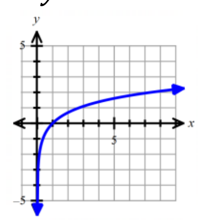
Exponential

$y = e^x$



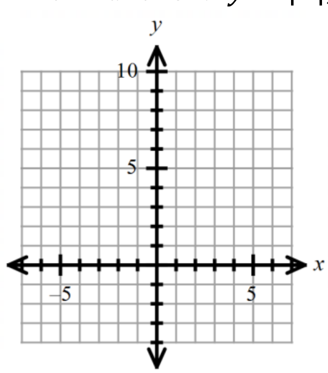
Logarithmic

$y = \ln x$

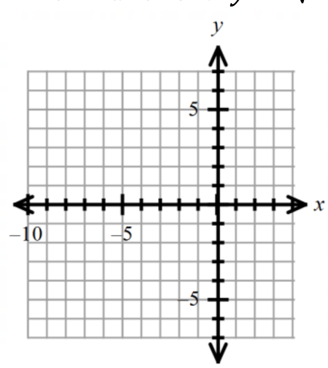


For #17 – 18, sketch each parent function. Then use transformations to sketch $g(x)$.

17) Parent function: $y = |x|$; $g(x) = 4|x| + 1$

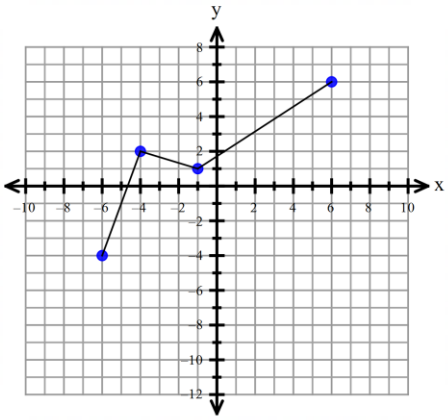


18) Parent function: $y = \sqrt[3]{x}$; $g(x) = -\sqrt[3]{x + 5}$

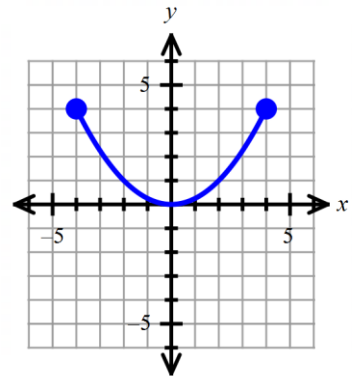


For #19 – 20, use the given graph of $y = f(x)$ to graph the described function $g(x)$ on the same graph.

19) $g(x) = -2f(x)$



20) $g(x) = -f(x) - 1$



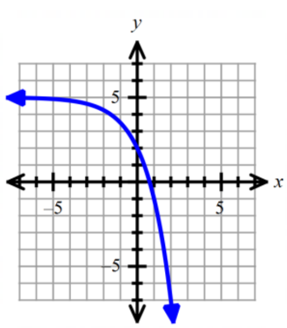
For #21 – 25, determine if each relation is a function or not.

21) $(-3, 2), (-1, 8), (4, 6), (-3, 5), (0, 7)$

22) $x^2 + y^2 = 25$

23) $y = -\sqrt{x+1}$

24)



25)